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SYSTEM MODELING AND STATISTICAL DATA PROCESSING.(U)

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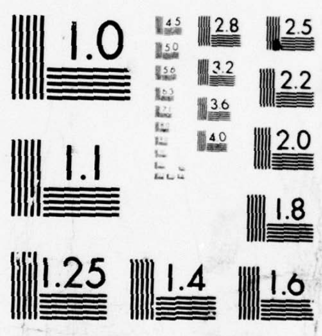
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SYSTEM MODELING AND STATISTICAL DATA PROCESSING

FINAL REPORT OF RESEARCH ACTIVITIES UNDER AFOSR CONTRACT

F44-620-74-C-0068

FOR THE PERIOD APRIL 1, 1974 - MARCH 31, 1979

The general research area is statistical modeling and signal processing, with emphasis on fast methods for signal estimation and detection. However, these studies have a mutually fruitful interaction with several other problems, especially in integral equations, scattering theory, multivariable systems, and more recently with network theory.

The scope of our research effort may be gathered from the following list of i) journal papers published during the report period, ii) papers accepted for publication, iii) papers submitted for review, iv) published conference papers and v) conference presentations (without published papers.

In addition to annual reports, a detailed review of work done during the first three years was submitted in mid-1977 and will not be reviewed here. As before, a good idea of the scope and progress of our continuing research effort can be gained from a perusal of papers published, papers to appear, and conference papers. In 1977-1978, we had one book and 8 journal papers published. There were 12 papers in some stage of revision. Of these 12, 10 appeared during 1978-1979 and 16 other papers moved into this category during this period. One other paper and a Springer-Verlag monograph also appeared. The journals in which they appeared are <u>IEEE Transactions on Information Theory</u>, on <u>Automatic Control</u>, on <u>Circuits and Systems</u>, on <u>Geoscience Electronics</u>, <u>The Annals of Statistics</u>, <u>Sankhyā</u>, <u>SIAM Review</u>, <u>Integral Equations and Operator Theory</u>, and in an Academic Press book edited by I. C. Gohberg and M. Kac. Ten conference papers were published , 5 at the 1978 IEEE Conference on Decision and Control, and one each at an International Symposium in Paris, an American Geophysical Union Chapman Conference in Pittsburgh, The Johns Hopkins Information Sciences Conference and an AFOSR Communication Theory Workshop in Provincetwon. Several other talks and seminars were presented, without formal written papers, at several conferences and seminars.

As in earlier years, this work has been well cited in the literature as can be seen from Science Citations Index.

Five Ph.D. theses were completed in the last two years by S. Kung (now an Assistant Professor at USC), A. Vieira (now in Brazil), J. Dobbins (now at Bell Laboratories), Sally Wood (now at Telesensor Systems), G. Verghese (now an Assistant Professor at MIT). Abstracts of these theses are attached, and along with the detailed list of publications, should give a better idea of the technical efforts under this contract.

PUBLISHED PAPERS

P-1 T. Kailath, A. Segall and M. Zakai, "Fubini-Type Theorems for Stochastic Integrals," Sankhyā, The Indian J. of Stat., vol. 40, Series A, 1978.

P-2 T. Kailath, A. Vieira and M. Morf, "Inverses of Toeplitz Operators, Innovations, and Orthogonal Polynomials," SIAM Review, vol. 20, no. 1, pp. 106-119, January 1978.

P-3 B. D. Anderson and T. Kailath, "Fast Algorithms for the Integral Equations of the Inverse Scattering Problem," Integral Equations and Operator Theory, vol. 1, no. 1, pp. 132-136, 1978.

P-4 M. Morf, A. Vieira and T. Kailath, "Covariance Characterization by Partial Autocorrelation Matrices," The Annals of Statistics, vol. 6, no. 3, pp. 643-648, 1978.

P-5 M. Morf, A. Vieira, D. T. Lee and T. Kailath, "Recursive Multichannel Maximum Entropy Spectral Estimation," IEEE Trans. on Geo-Science Electronics, vol. GE-16, no. 2, pp. 85-94, April 1978.

P-6 T. Kailath, B. Levy, L. Ljung and M. Morf, "Fast Time-Invariant Implementations of Gaussian Signal Detectors," IEEE Trans. on Information Thy., vol. IT-24, no. 4, pp. 469-477, July 1978.

P-7 B. Friedlander, T. Kailath, M. Morf and T. Kailath, "Extended Levinson and Chandrasekhar Equations for General Discrete-Time Linear Estimation Problems," IEEE Trans. on Automatic Control, vol. AC-23, no. 4, pp. 653-659, August 1978.

P-8 M. Morf, B. Levy and T. Kailath, "Square-Root Algorithms for the Continuous-Time Linear Least-Square Estimation Problem," IEEE Trans. on Automatic Control, vol. AC-23, no. 5, pp. 907-911, October 1978.

P-9 R. R. Bitmead, S-Y. Kung, B. D. Anderson and T. Kailath, "Greatest Common Divisors Via Generalized Sylvester and Bezout Matrices," IEEE Trans. on Automatic Control, vol. AC-23, no. 6, pp. 1043-1047, December 1978.

P-10 T. Kailath, L. Ljung and M. Morf, "Generalized Krein-Levinson Equations for Efficient Calculation of Fredholm Resolvents of Nondisplacement Kernels," Topics in Functional Analysis, Advances in Math. Supplementary Studies, vol. 3, pp. 169-184, 1978.

P-11 G. Verghese and T. Kailath, "A Further Note on Backwards Markovian Models," IEEE Trans. on Information Thy., vol. IT-25, no. 1, pp. 121-124, January 1979.

PUBLISHED BOOKS

T. Kailath, Lectures on Linear Least-Squares Estimation, CISM Courses & Lectures No. 140, International Centre for Mechanical Sciences, New York: Springer-Verlag, 1978.

ACCEPTED PAPERS TO APPEAR

A-1 G. Verghese, P. Van Dooren and T. Kailath, "Properties of the System Matrix of a Generalized State-Space System," International J. of Control, to appear.

A-2 G. Verghese and T. Kailath, "Comments on 'On Structural Invariants and the Root-Loci of Linear Multivariable Systems'," International J. of Control, to appear.

A-3 B. D. Anderson and T. Kailath, "Passive Network Synthesis via Dual Spectral Factorization," IEEE Trans. on Circuits & Systems, to appear.

A-4 B. Levy, L. Ljung and M. Morf, "The Factorization and Representation of Operators in the Algebra Generated by Toeplitz Operators," SIAM J. on Appl. Math., to appear.

A-5 B. Friedlander, M. Morf, T. Kailath and L. Ljung, "New Inversion Formulas for Matrices Classified in Terms of their Distance from Toeplitz Matrices," Linear Algebra and Its Applications, to appear.

A-6 B. D. Anderson and T. Kailath, "Forwards and Backwards Models for Finite-State Markov Processes," J. of Applied Probability, to appear.

A-7 A. Segall and T. Kailath, "Martingales in Nonlinear Least-Squares Estimation Theory," Nonlinear Filtering, ed. by E. Stear, Marcel Dekker Publishing Co., to appear.

A-8 T. Kailath, "Sigma-Fields, Martingales, Stochastic Integrals and All That (Almost Surely)," Proc. IEEE, to appear.

A-9 M. Morf, J. R. Dobbins, B. Friedlander and T. Kailath, "Square-Root Algorithms for Parallel Processing in Optimal Estimation," Automatica, vol. 15, 1979.

A-10 T. Kailath, S-Y. Kung and M. Morf, "Displacement Ranks of Matrices and Linear Equations," J. of Math. Analysis and Applications, to appear.

A-11 T. Kailath, S-Y. Kung and M. Morf, "Displacement Ranks of a Matrix," Bulletin American Math. Society, 1979.

A-12 S-Y. Kung and T. Kailath, "Fast Algorithms for the Minimal Design Problem," Automatica, to appear.

A-13 M. Morf, E. Verriest, J. R. Dobbins and T. Kailath, "Square-Root Algorithms for Model Sensitivity Analysis," Automatica, to appear.

A-14 B. Friedlander and T. Kailath, "Scattering Theory and Linear Least Squares Estimation, Pt. III: The Estimates," IEEE Trans. on Automatic Control, to appear.

A-15 G. Verghese, B. Levy and T. Kailath, "Generalized State-Space Systems," IEEE Trans. on Automatic Control, to appear.

A-16 B. Levy, T. Kailath, L. Ljung and M. Morf, "Fast Time-Invariant Implementations for Linear Least-Squares Smoothing Filters," IEEE Trans. on Automatic Control, to appear.

SUBMITTED PAPERS

S-1 T. Kailath, L. Ljung and M. Morf, "Recursive Input-Output and State-Space Solutions for Continuous-Time Linear Estimation Problems," submitted to IEEE Trans. on Automatic Control, 1978.

S-2 G. Verghese and T. Kailath, "Rational Matrix Structure," submitted to IEEE Trans. on Automatic Control, 1979.

CONFERENCE PAPERS

C-1 M. Morf, J. Dobbins, J. Newkirk and T. Kailath, "Square-Root Doubling for Steady State Riccati Solution: A Viable Method?," Conference on Information Sciences & Systems, Johns Hopkins University, Baltimore, MD, March 1978.

C-2 T. Kailath, "Some Alternatives in Recursive Filtering," Chapman Conference on Applications of Kalman Filtering Theory and Technique to Hydrology, Hydraulics, and Water Resources, Univ. of Pittsburgh, Pittsburgh, PA, May 1978.

C-3 M. Morf and D. Lee, "Recursive Spectral Estimation of Alpha-Stationary Processes," Proc. Rome Air Development Center, Spectrum Estimation Workshop, Griffiss AFB, New York, pp. 97-108, May 1978.

C-4 T. Kailath, "Forwards and Backwards Markovian Models for Second-Order Processes," Proc. AFOSR Workshop in Communication Theory and Applications, pp. 2-3, Provincetown, MA, September 1978.

C-5 T. Kailath, A. Vieira and M. Morf, "Orthogonal Transformation (Square-Root) Implementations of the Generalized Chandrasekhar and Generalized Levinson Algorithms," International Symposium on Analysis and Optimization of Systems, IRIA, Paris, December 1978.

C-6 G. Verghese, B. Lévy and T. Kailath, "Generalized State-Space Systems," Proc. 1978 IEEE Conference on Decision & Control, San Diego, CA, pp. 518-520, January 1979.

C-7 S. Kung and T. Kailath, "Some Notes on Valuation Theory in Linear Systems," Proc. 1978 IEEE Conference on Decision & Control, San Diego, CA, pp. 515-517, January 1979.

C-8 B. Lévy, T. Kailath, L. Ljung and M. Morf, "Fast Time-Invariant Implementations for Linear Least-Squares Smoothing Filters," Proc. 1978 IEEE Conference on Decision & Control, San Diego, CA, pp. 1156-1159, January 1979.

C-9 P. Van Dooren, G. Verghese and T. Kailath, "Properties of the System Matrix of a Generalized State-Space System," Proc. 1978 IEEE Symposium on Decision & Control, San Diego, CA, pp. 173-175, January 1979.

C-10 M. Morf and D. Lee, "Recursive Least Squares Ladder Forms for Fast Parameter Tracking," Proc. 1978 IEEE Conference on Decision & Control, San Diego, CA, pp. 1362-1367, January 1979.

Multivariable and Multidimensional Systems: Analysis and Design
June 1977

S. Kung

ABSTRACT

This thesis contains two parts. The first part presents some topics in the analysis and design of multivariable (multi-input, multi-output) systems. The second part contains results that provide a foundation for a comprehensive state-space and algebraic study of systems described by two or more independent variables, which are termed multidimensional systems.

In the first part of this thesis, we give new algorithms for determining the greatest common divisor (GCD) of two polynomial matrices as well as solving minimal design and partial realization problems. Our approach will demonstrate the power of using certain polynomial echelon matrix fraction descriptions, which depend closely upon the fundamental concepts of column-reduced matrices and minimal bases of vector spaces over the rational field.

The time-invariance of the systems we deal with ensures a certain shift-invariance structure in some basic matrices, such as the generalized resultant matrix and the Hankel matrix that arise in many linear systems problems. Combining these invariance properties with the polynomial echelon form, we develop a variety of fast algorithms for GCD extractions, minimal design problems, and minimal partial realizations.

In the second part of this thesis, we successfully extend some existing one-dimensional results, such as GCD extractions, coprimeness tests, and matrix fraction descriptions to the two-dimensional case.

We also discuss some results that appear to be unique to multidimensional systems, such as the existence and uniqueness of primitive factorizations, and the general factorizations of two-dimensional polynomial matrices. It appears to be the first time that two dimensional polynomial matrix theory has been studied to such an extent.

We also present several new results on two-dimensional state-space representations, including comparisons of various existing models, methods for circuit realizations using a minimal number of dynamic elements, and a new technique for two-dimensional digital filter hardware implementations.

MATRIX ORTHOGONAL POLYNOMIALS, WITH APPLICATIONS TO AUTOREGRESSIVE MODELING AND LADDER FORMS

Augusto Cesar Gadelha Vieira, Ph. D.

Stanford University

In this work we obtain some results related to modeling and filtering of multivariate time series. A framework for the development of such results is provided by the theory of orthogonal polynomials on the unit circle. This theory and its applications, such as in stochastic and statistical problems, are by now well established for the scalar case. In chapter I we give an account of the theory of matrix orthogonal polynomials with some of their most useful properties. These seem to be less known even though some results in this context have already been reported in the literature. This subject has great interest by itself. For the sake of generality, we assume an indefinite weighting matrix, specializing for the non-negative definite case when needed.

The orthogonal polynomials obey well known recursions that play a central role in our work. These are the same as the equations obtained in the problem of fitting an autoregressive model to a given segment of a covariance function. By considering a normalized form of these recursions we show in chapter II that a covariance function can be uniquely specified by a sequence of "partial correlation" or "reflection" matrix coefficients, each of which has singular values of magnitude less than one. This result is used in the following chapters.

Chapter III explores some connections of the previous chapters with network theory. The above mentioned recursions can be regarded as performing a coprime factorization of a certain matrix function, such a factorization being equivalent to the Darlington synthesis procedure in network theory. This interpretation enables us to generalize these recursions to produce

autoregressive-moving average models of a covariance function. Closely related to this is the theory of the multiplicative representation of a J-contractive matrix function as established by Potapov.

The autoregressive fitting procedure mentioned above has led to a new method of spectral estimation based on direct estimation of the reflection coefficients, generally known as (Burg's) maximum entropy method. There have been many succesful applications of this technique, e.g. in the analysis of seismic and speech signals. However, satisfactory multivariate (or multichannel) extensions of this technique have been obtained only recently. In chapter IV we give one such extension, using the result of chapter II.

The reflection coefficients are usually estimated by cross-correlating (using sample averages) the so-called forward and backward residuals. This method is most suited for batch data processing. In many applications on-line implementations are desired, e.g. in speech processing. In this case the use of instantaneous sample averages, or some slight modification thereof, has been proposed in the literature. However, these approximations tend to converge very slowly to the batch sample averages. This is partially due to the fact that no global error criteria is minimized. As an alternative we present in chapter V a new class of ladder forms. The reflection coefficients of these forms are recursively computed from the data, such that their impulse response is equal to the exact (global) least-squares predictor. These new ladder forms provide alternatives that have some advantages over the usual maximum entropy methods even when batch processing is used. Experimental results seem to confirm these expectations.

S. Wood

A System Theoretic Approach to Image Reconstruction
May 1978.

ABSTRACT

The problem of extracting all of the information from each measurement used for image reconstruction in computerized tomography becomes important when only a limited number of measurements are available or when the measurements have a significant variance. These conditions can occur when dosage is lowered, in limited angle applications, or when the object to be reconstructed is moving periodically so that only a limited number of synchronized measurements are valid. In such cases algorithms which take into account the stochastic nature of the measurements and include a priori information generally perform much better than deterministic methods even though the deterministic algorithms do perform well in common applications with abundant low noise measurements.

In this thesis the linear minimum variance estimator is applied to emission and transmission tomography. Both information filter and covariance filter forms are discussed as well as recursive square root implementations. The influence on mean squared error of variations in dosage and measurement geometry is examined in terms of singular values of the projection matrix.

Standard deterministic algorithms are analyzed in a stochastic context. For each algorithm the implicit assumptions that would make it equivalent to the minimum variance estimator are derived. This analysis results in the specification of otherwise arbitrary parameters in the algorithms. New approximate algorithms based on specific statistical assumptions are derived.

Symmetries of the measurement acquisition arrangement are used to achieve an order of magnitude savings in computation time and storage required for implementation of the minimum variance estimator. Three possible implementation strategies are discussed. The specific symmetries required do not exclude the limited angle application.

Simulations of expected squared errors demonstrate a substantial improvement when the minimum variance estimator is used instead of approximate methods in both low and high noise cases. The new approximate algorithms give consistently better results than ART in medium and high noise cases. Simulated reconstructions of a phantom give squared error results consistent with the mean squared error predictions. In addition, features of the phantom which are visible in the minimum variance estimator reconstruction cannot be seen in the approximate method reconstructions.

The minimum variance estimator is shown in this thesis to offer substantial improvement in performance over more classical reconstruction techniques, particularly when dosage must be lowered or a full set of measurements is not available. In cases such as these, computationally efficient implementations of the minimum variance estimator such as those discussed here should prove to be diagnostically significant.

G. Verghese

Infinite-Frequency Behaviour in Generalized Dynamical Systems
December 1978.

ABSTRACT

The dynamical systems considered in this thesis are systems of ordinary, linear, time-invariant differential equations that, starting from some initial time, relate the behaviour of a set of internal system variables to that of control inputs and observation outputs for the system. In particular, we are concerned with those dynamical systems that, with control inputs fixed at zero, are capable of impulsive motions at the initial time if given arbitrary initial conditions, motions that may be formally treated as infinite-frequency free-response modes. Such systems we term generalized dynamical systems.

Their behaviour is essentially accounted for by the fact that their describing equations imply certain constraints on the associated variables that may not be satisfied if arbitrary initial conditions are imposed; the impulses are then a consequence of the transition to consistent variable values. The reason that a framework for treating such systems has not emerged prior to our work may now, in view of the preceding interpretation, be explained: in the extensive literature on dynamical systems it is tacitly assumed, virtually without exception, that the systems considered have existed prior to the initial time of analysis; the resulting constraints on the initial conditions then guarantee that

no impulsive behaviour occurs.

Our framework becomes important for systems formed at the initial time, as a result of perhaps switching or component failure in other systems. It also yields new and interesting results for systems which, if formed at the initial time, could exhibit impulsive behaviour; this point of view is vital when considering, for example, how properties of subsystems are reflected into those of composite systems formed by interconnecting them. It is conjectured that our theory will, in addition, be useful in treating systems that are idealized limits of other ('singularly perturbed') systems that do not impose constraints (or that impose different constraints) on the associated variables. Besides such applications, our development leads to new results on, and insights into the behaviour at infinite frequencies of even those dynamical systems (which we term regular dynamical systems) that exhibit no impulsive free-response modes.

A tutorial, but yet somewhat novel treatment of the pole-zero structure of rational, frequency-domain, transfer-function matrices, at finite and infinite frequencies, forms the background for our analysis. Pole-zero structure is studied via both general decompositions to diagonal matrices and transformations to so-called column- (or row-) reduced matrices (with certain new results on rational bases for rational vector spaces emerging from our study of these reduced matrices).

A description of the detailed structure of the infinite-frequency or impulsive modes of an undriven system is then obtained, apparently for the first time. A generalized order that accounts for the degrees of freedom associated with all the modes, impulsive and non-impulsive, is defined. The coupling of the impulsive modes to the system input and

and output is characterized by definitions of controllability and observability for these modes. Much of the theory for regular dynamical systems is thereby extended to generalized dynamical systems.

More detailed and extensive results are obtained for the special case of generalized state-space systems, which comprise first-order differential equations coupled with algebraic constraints, and form the simplest interesting example of generalized dynamical systems. For such systems a useful class of equivalence transformations is identified, and used, for example, to display the unobservable and/or uncontrollable impulsive modes.

Covariance Factorization Techniques for Least Squares Estimation
January 1979

J. R. Dobbins

Abstract

Square root, least squares estimation algorithms are examined from the viewpoint that square root arrays are arrays expressing relevant problem random variables in terms of basis sets of orthonormal (identity covariance) random variables. With this viewpoint, projection-theorem arguments can be expressed directly in the square root arrays, giving simple, geometric derivations of square root algorithms. A unifying framework is provided for previously proposed square root algorithms (and square-root-free factorization algorithms), and several new square root estimation procedures are derived.

An inversion duality relating covariance and information forms is demonstrated to be more widely applicable than the standard duality. A compact explanation for previously reported techniques of a posteriori determination of the effects of neglecting bias parameters in the model and of computational reduction when subsets of the estimates are conditionally independent is given in terms of the relationship among orthonormal basis variables induced by triangular square root arrays. Square root versions of all major smoothing algorithms are derived; several of these square root versions have not been published previously.

A new set of square root algorithms is proposed in which a good deal of the computation may be performed in parallel by different processors with very little inter-processor communication. Even when only a single processor is available, the approach underlying these algorithms offers computational reductions when covariances are needed at relatively few few time points in an interval. This is especially true when the model

of the observed process has time invariance properties. One specialization of particular interest is a square root doubling algorithm for computing the steady state solution of time invariant, matrix Riccati equations. This algorithm computes square roots of $P(t)$, $P(2t)$, $P(4t)$, ... on successive iterations, where P is the matrix Riccati variable.

Square root techniques for determining the actual error covariance of a state estimate based on an incorrect model are extended. The ease with which either of two previously proposed techniques may be used with either the covariance or the information form of the estimator is enhanced. The two techniques are compared, and some guidance is provided as to when the use of each is advantageous. Square root error analysis is extended to the smoothing problem.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Square root, least squares estimation algorithms are examined from the viewpoint that square root arrays are arrays expressing relevant problem random variables in terms of basis sets of orthonormal (identity covariance) random variables. With this viewpoint, projection-theorem arguments can be expressed directly in the square root arrays, giving simple, geometric derivations of square root algorithms. A unifying framework is provided for previously proposed square root algorithms (and square-root-free factorization algorithms), and several new square root estimation procedures are derived. (continued)

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20. Abstract (continued)

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A new set of square root algorithms is proposed in which a good deal of the computation may be performed in parallel by different processors with very little inter-processor communication. Even when only a single processor is available, the approach underlying these algorithms offers computational reductions when covariances are needed at relatively few few time points in an interval. This is especially true when the model of the observed process has time invariance properties. One specialization of particular interest is a square root doubling algorithm for computing the steady state solution of time invariant, matrix Riccati equations. This algorithm computes square roots of P(t), P(2t), P(4t),... on sucessive iterations, where P is the matrix Riccati varible.

Square root techniques for determining the actual error covariance of a state estimate based on an incorrect model are extended. The ease with which either of two previously proposed techniques may be used with either the covariance or the information form of the estimator is enhanced. The two techniques are compared, and some guidance is provided as to when the use of each is advantageous. Square root error analysis is extended to the smoothing problem.